Professor Marilyn Carlson discusses a programme that has proven effective in improving teaching of precalculus concepts, as well as students’ chances of success in more advanced mathematics.

Why is precalculus of such fundamental importance to student development, and how has the Pathways Project sought to improve precalculus understanding?

Precalculus mathematics lays the foundation for learning calculus, and for modelling quantitative relationships in the sciences and engineering. A student who emerges from precalculus with weak understandings of key ideas, such as rate of change and function, is not prepared to learn calculus and be successful in STEM fields.

With this in mind, the Pathways Project developed the Precalculus Concept Assessment (PCA), a 25-item multiple-choice instrument, to assess student understanding of the foundational ideas and reasoning abilities of precalculus mathematics. The exam has now been administered to over 10,000 students and has consistently shown that those who score lower than 12 out of 25 are not likely to succeed in calculus.

In addition to the PCA, over the past 20 years my research has uncovered foundational ways of thinking that support students in learning key ideas of precalculus. This knowledge has driven the development and refinement of the Pathways student curriculum and Pathways Professional Development model for secondary mathematics teachers. Every component of Pathways is grounded in and refined by qualitative studies of what is involved in understanding and learning key precalculus ideas. These studies produced a new theory of learning and teaching precalculus ideas, and also informed the development and refinement of precalculus curriculum and teacher tools.

What findings emerged from the PCA?

In the early stages, PCA data revealed that the understanding of both students and secondary mathematics teachers of key ideas of precalculus was weak. During the first five years of Pathways Professional Development, we documented large gains in teachers on the PCA, but very low or no student gains. This led us to conduct qualitative studies with specific mathematics teachers. Six case studies revealed that the deep understandings teachers exhibited in a workshop setting were very fragile. When visiting their classrooms, we witnessed teachers often reverting back to old ways of showing students methods for obtaining answers. This led us to conjecture that teachers need the support of meaningful and coherent student curriculum and instructional tools. The supports should also be helpful for solidifying teacher understandings of key ideas, and enable them to build more connections among ideas when teaching.
Calculating success

The Pathways Project at Arizona State University offers an innovative suite of tools for assessing understanding of primary precalculus concepts, increasing curriculum quality and teaching effectiveness. The initiative improves students’ success in calculus and, ultimately, STEM.

**WITHIN THE NEXT decade, if the US is to maintain its dominant position in the science and technology sectors, or at the very least to remain competitive in global markets, it will need many more workers skilled in STEM subjects than are currently available to fill its technical jobs. As a result, there is a nationwide imperative to improve STEM teaching, increase STEM student enrolment and retention, and ensure that universities successfully deliver the high levels of skills required for the marketplace.**

**A NOVEL APPROACH TO STEM SUCCESS**

At Arizona State University (ASU), an initiative has been underway over the last 10 years to develop better understanding of the primary concepts that are preparatory for students to succeed in and understand calculus at both schools and universities. The endeavour aims not only to ensure success in later calculus studies at a higher level and so encourage more students to persist in studying mathematics and science, but also to significantly improve student ability and confidence in solving novel problems. The Pathways Project seeks to develop deeper understanding of key precalculus ideas – such as quantity, variable, function, rate of change, exponential growth, angle measurement, and trigonometric functions – in both students and teachers, and is based on in-depth research into learning processes, teacher knowledge and teaching effectiveness. The Pathways curriculum and teacher resources were designed and refined through iterative research cycles. As we continue to understand more about effective ways to achieve greater learning and teaching, this knowledge will inform future Pathways revisions.

The usual approach to designing interventions for improving STEM teaching and learning is to draw together a committee of well-meaning people with relevant experience who then debate and propose action. Essentially, the process is unscientific and the product most often will not meet the need, as Professor Marilyn Carlson, Pathways Principal Investigator and member of the ASU Mathematics and Statistics faculty highlights: “It doesn’t make sense that most of the science and mathematics curriculum being used in the US has no basis in research on learning or understanding the key ideas that are its focus. There is little or no attention given to achieving measurable gains in student learning of key concepts and STEM practices”.

In 2010, Carlson and her collaborator Dr Michael Oehrtman developed the Precalculus Concept Assessment (PCA), a test that clinically validated as a reliable indicator of fundamental understanding of precalculus mathematics. The PCA has since been administered to thousands of students, as well as mathematics teachers and university faculty members. As a result of early tests administered to mathematics teachers, Carlson and her co-investigators realised that, while there was an urgent need for much better precalculus teaching practices, the teachers themselves often lacked the knowledge required to apply them effectively. "It was particularly surprising that many secondary school teachers, with first degrees in mathematics, did not have a strong understanding of ideas such as exponential growth, function composition, function inverse, function notation and trigonometric functions," Carlson observes. She therefore launched the Pathways Project, under the umbrella of which research has been conducted into all aspects of learning and teaching of precalculus concepts and approaches to problem solving. Pathways now provides a model system that spans teacher professional development, precalculus curriculum development, supplementary teaching resources and learning aids.

**LEARNING THROUGH PATHWAYS**

From the students’ perspective, the Pathways learning approach requires them to endeavour to apply four main behaviours: firstly, students engage in ‘sense making’, which involves analysing, conceptualising and expressing (perhaps pictorially) a mathematical problem in ways that enable them to visualise the problem statement, and understand both the known and unknown quantities involved; secondly, they are asked to explain their thinking and demonstrate the logic that underpins their reasoning; thirdly, they are challenged to develop meaningful formulas, graphs and tables to represent the relationships between the quantities described in the problem context;
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Marilyn Carlson is Professor in the School of Mathematical and Statistical Sciences and Director of Research Innovations in Mathematics Education at Arizona State University. She has over 70 research publications in mathematics education, and is a frequent national and international speaker on issues related to knowing, learning, and teaching mathematics. Carlson has received over US $20 million in funding from the National Science Foundation for her research and outreach work with secondary and undergraduate mathematics instructors. Her current NSF grant, Project Pathways, is working in over 20 secondary schools, community colleges and universities to support improvements in curriculum and student learning in precalculus and calculus. She was the Coordinator of the Mathematical Association of America (MAA) Special Interest Group for Research in Mathematics Education, and edited the MAA Volume, Making the Connection: Research to Practice in Mathematics Education. She was granted an NSF CAREER award to study student learning in calculus. In 2007, she was presented the MAA Selden Award for Research in Undergraduate Mathematics Education, and in 2013 she received the Outstanding Doctoral Mentor award at Arizona State University.

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With these principles established, students work in groups around a whiteboard to undertake an investigation composed of a sequence of thought provoking questions. These are designed and ordered to encourage the students to construct meanings and connections that lead to understanding and the application of ideas to solve unfamiliar problems. During the investigation, the preparation of responses to a specific cluster of questions is timed to assure that students are active and productive in their collaborative groups.

Within each group, a designated member documents the process and the solution. All students are required to understand these aspects in detail, as any of them may be called upon to describe the solution and the ways of thinking that led to it. “Once the groups have constructed their written products, the teacher asks three to four groups to explain their solutions to the class,” Carlson elaborates. “This typically leads to high quality discourse among the group members as they work to ensure that everyone is prepared to provide a coherent explanation.” Students from the other groups can then ask questions for clarification to which only the presenter may initially respond. After that, other members of the presenting group are given the opportunity to add further clarifications. The teacher can also ask questions of the group to ensure that the meanings and connections are fully and clearly articulated.

TEACHING THROUGH PATHWAYS

The Pathways model of effective teaching is based on established techniques and the results of case studies carried out with mathematics teachers in which their knowledge, curriculum and classroom practices were closely evaluated. These studies consistently revealed that the most critical variable for effective teaching is the teachers’ understanding of a course’s ideas and how they are connected. This enables them to evaluate their students’ thinking, pose meaningful questions and provide coherent and conceptually orientated explanations and wrap-up lectures. The Pathways Professional Development programme trains teachers in how to interpret and react to student thinking while teaching key concepts. The programme is delivered over three years, starting with a three-to-five day workshop followed by a monthly workshop for the first year, during which the teachers apply the Pathways approach in the classroom.

Teachers are supplied with a full package of materials, including instructional sequences that facilitate the learning of key ideas, as well as teaching notes and sample solutions, such as desirable and undesirable ways of thinking about a problem or approaching complex tasks. They are also given investigation frameworks with accompanying homework assignments; interactive PowerPoint slideshows aligned with the investigations to assist the teacher in leading concept-orientated discussions; and computer animations designed to support the development of critical reasoning abilities and connections. Furthermore, the materials include examinations and other assessment devices; and supplementary resources such as videos and alternative investigation solutions.

FUTURE PATHWAYS

Over the last five years, the Pathways model has been implemented with more than 400 mathematics teachers and 20,000 students. During this same time frame, the model has shown dramatic improvements in both the teachers’ and students’ knowledge of key precalculus concepts. Before Pathways, the best mean PCA score for a class of students at 40 different colleges and universities was 9.1 out of 25, when 13 is the minimum readiness indicator for success with calculus; however, after being taught by teachers with Pathways training and using Pathways materials, the mean score of an equivalent set of students was 16.2.

The Pathways Project’s achievement with precalculus has resulted in it being extended to Pathways to Algebra I and Algebra II. The Algebra II professional development and curriculum was implemented in nine schools (with a total of 45 teachers) in 2012-13, with all schools having significant gains in student learning of the course’s key ideas. The Algebra I curriculum and professional development has been developed and is being piloted in three schools during the current 2013-14 academic year. Judging by the programme’s positive impact so far, the Pathways Project is surely on the road to being a successful model for improving the quality of teaching and student learning in precalculus level courses.